Data-Driven Robust Controller for Artificial Pancreas

CyberCardia Meeting @Stony Brook 4/22/2016
Data-Driven Robust Controller for Artificial Pancreas

Motivation: Artificial Pancreas

Diabetes patient
• Lack of insulin production
• Cannot control blood glucose level
• ↓ insulin -> ↑ glucose

Artificial pancreas
• Monitor glucose level
• Inject insulin if needed
Data-Driven Robust Controller for Artificial Pancreas

Related work: Controller

- [Hovorka 04] – Nonlinear model predictive control of glucose concentration in subjects with type 1 diabetes
- [Dalla 07] – Model Predictive Control of Type 1 Diabetes: An in Silico Trial

Model Predictive Controller

- Predictions based on the model and control accordingly

Why?

- Anticipate future events – predictive power
- Optimize a finite time-horizon, only implementing current timeslot
- Model time delay
Data-Driven Robust Controller for Artificial Pancreas

Challenge: Robustness

Robustness against uncertainty
• Event uncertainty
• Model uncertainty

More on these later!
Optimization & Verification

• Previous Talk:
  • Composition of Heart Model and Device (controller) Model
  • Property expressed in Logic
  • Verification

• Model-Based Falsification of an Artificial Pancreas Control System
  • Composition of Physiological insulin-glucose regulatory Model and Device (controller) Model
  • Property expressed in Logic
  • Falsification tools to find the least robust trace

• This work
  • Patient’s insulin-glucose model
  • Develop a controller to achieve some objective (optimization according to some “cost” function)
Artificial Pancreas & Taxi dispatcher

• Components: Model Predictive Control + Data + Robustness

• Previous works on Taxi Dispatch Problem [Fei Miao, Shan Lin, et al]
  • Robust Taxi Dispatch under Model Uncertainties (CDC 2015)
  • Data-Driven Robust Taxi Dispatch under Demand Uncertainties (TCST 2016 / ICCPS 2016 Poster)
  • Taxi Dispatch with Real-time Sensing Data in Metropolitan Areas: A Receding Horizon Control Approach (ICCPS 2015 / TASE 2016)
Data Driven Robust Taxi Dispatch under Demand Uncertainties

- **Spatial-temporally correlated demand uncertainties**
- **Sensing data of taxis**
- **Predictive Demand**
- **Robust MPC for Global Taxi Dispatch with Demand Uncertainty Set**
- **Current and anticipated future cost**
- **Balanced supply and service fairness**
- **Minimum idle distance to meet demand**
Taxi Dispatch as Optimization

\[
J = \sum_{k=1}^{\tau} (J_D(X^k) + \beta J_E(X^k, r^k))
\]

\[
= \sum_{k=1}^{\tau} \sum_i \left( \sum_j X^k_{ij} W_{ij} + \frac{\beta r^k_i}{(1_n^T X^k_i - X^k_i 1_n + L^k_i)^\alpha} \right)
\]

s.t. \((L^{k+1})^T = (1_n^T X^k - (X^k 1_n)^T + (L^k)^T) P^k,\)

\(1_n^T X^k - (X^k 1_n)^T + (L^k)^T \geq 1,\)

\(X^k_{ij} W_{ij} \leq m X^k_{ij},\)

\(X^k_{ij} \geq 0, \quad i, j \in \{1, 2, \ldots, n\}.\)

\(11\)
Data Driven Robust Artificial Pancreas under Patient Uncertainties

Patient-specific uncertainties

Sensing data of glucose

Model parameters

Robust MPC for Insulin Injection with Uncertainty Set

Meet patient safety requirement

Small variance of glucose level

Fast response to meal and exercise events

Glucose level

Real Time Control

Insulin injection

Insulin pump

Glucose level

Control Algo Device
Artificial Pancreas Controller as Optimization

- **Cost function** $J$ depends on glucose level: 
  \[ J(<g_1, ..., g_T>) \]

- Insulin-glucose model $M$ with uncertainty $D$: 
  \[ <g_1, ..., g_T> = M(<i_1, ..., i_T>, D) \]

- Decision problem: 
  \[ <i_1^*, ..., i_T^*> = \min_{<i_1, ..., i_T>} J(M(<i_1, ..., i_T>, D)) \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$i_t$</td>
<td>Insulin level at time $t$</td>
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<tr>
<td>$g_t$</td>
<td>Glucose level at time $t$</td>
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<tr>
<td>$M$</td>
<td>Patient model</td>
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<td>$D$</td>
<td>Model uncertainty</td>
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<td>$J$</td>
<td>Cost function</td>
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<td>$T$</td>
<td>Prediction horizon</td>
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Insulin amount
Artificial pancreas model

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<tr>
<th>Model Name</th>
<th>Type</th>
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<td>Hovorka 04</td>
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<td>Dalla 07</td>
<td>Nonlinear</td>
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</table>
Model – Glucose & Insulin subsystem

- Gut absorption: $U_G$
- Insulin absorption: $U_l/V_l$

Accessible glucose:
- $Q_1 \rightarrow Q_2$
- $x_1 \rightarrow x_2 \rightarrow x_3$

Non-accessible glucose:
- $G = Q_l/V_G$
- $F^{c_0}Q_l/(GV_G)-F_R$

Insulin amount:
- $k_{a1}, k_{a2}, k_{a3}, k_{b1}, k_{b2}, k_{b3}$
Model – Insulin action subsystem

Insulin action on glucose transport
Insulin action on glucose disposal
Insulin action on glucose production

Glucose subsystem: $Q_1, Q_2$
Insulin subsystem: $I$
Model – External factors

\[ U_L(0) \propto D_L \]

where \( D_G \) is amount of carbohydrates digested.

\[ U_{G(t)} \propto D_G \]
Model – Mathematics

\[ U_G(t) = \frac{D_G A_G t e^{-t/t_{max,G}}}{t^2_{max,G}} \]

\[ \frac{dQ_1(t)}{dt} = -\left( \frac{F_{01}^c}{V_G G(t)} + x_1(t) \right) Q_1(t) + k_{12} Q_2(t) - F_R + U_G(t) + EGP_0[1 - x_3(t)] \]

\[ \frac{dQ_2(t)}{dt} = x_1(t) Q_1(t) - [k_{12} + x_2(t)] Q_2(t) y(t) G(t) = Q_1(t)/V_G \]

\[ \frac{dI(t)}{dt} = \frac{U_I(t)}{V_I} - k_e I(t) \]

The model represents three actions of insulin on glucose kinetics

\[ \frac{dx_1}{dt} = -k_{a1} x_1(t) + k_{b1} I(t) \]

\[ \frac{dx_2}{dt} = -k_{a2} x_2(t) + k_{b2} I(t) \]

\[ \frac{dx_3}{dt} = -k_{a3} x_3(t) + k_{b3} I(t) \]
Parameters

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<th>$k_{12}$ (min$^{-1}$)</th>
<th>$k_{b1}$</th>
<th>$k_{b2}$</th>
<th>$k_{b3}$</th>
<th>$S_{IT}^{f}$ ($10^{-4}$ × min$^{-1}$)</th>
<th>$S_{ID}^{f}$ ($10^{-4}$ × per mU/L)</th>
<th>$F_{01}$ (μmol·kg$^{-1}$·min$^{-1}$)</th>
<th>$V$ (l/kg)</th>
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Disposal of glucose due to insulin
Next steps

• Data – to construct uncertainty set
  • Patient uncertainty: physiological parameters of patients
  • Event uncertainty: meal / exercise event

• Optimization
  • Recall \( \min_{<i_1, \ldots, i_T>} J(M(<i_1, \ldots, i_T>, D)) \)
  • Transform to multi-stage optimization:
    \[
    \min_{i_1} \max_{d_1} \min_{i_2} \max_{d_2} \ldots \min_{i_T} \max_{d_T} J(M(<i_1, \ldots, i_T>, D))
    \]
Challenge

• Modeling and Constructing uncertainty
  • Representation
  • Learning from data

• Optimization tractability