Closed-Loop Robust Control for an Artificial Pancreas under Meal and Exercise Uncertainty

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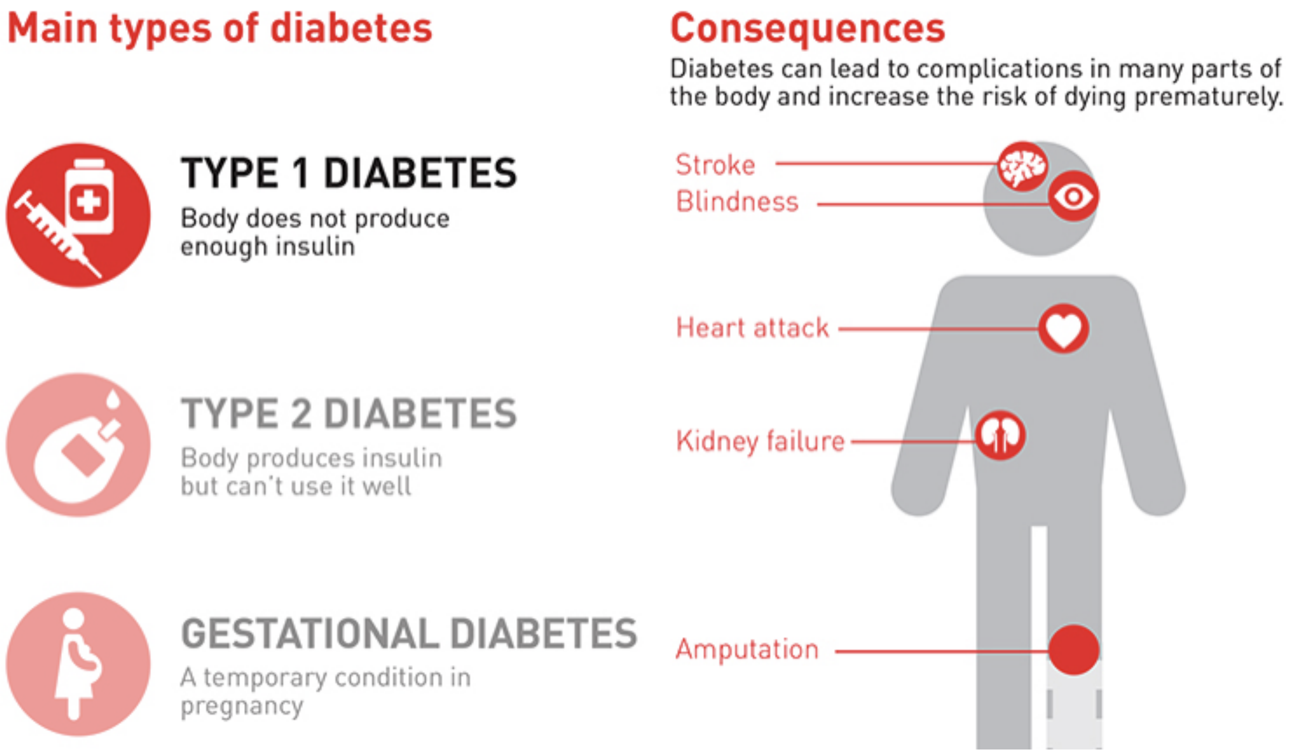
Joint work with: Shan Lin, Scott Smolka, Kin Sum Liu

CyberCardia meeting at UPenn, 14 Apr 2017

Diabetes



WHO Global report on diabetes, 2016

Type 1 Diabetes therapy

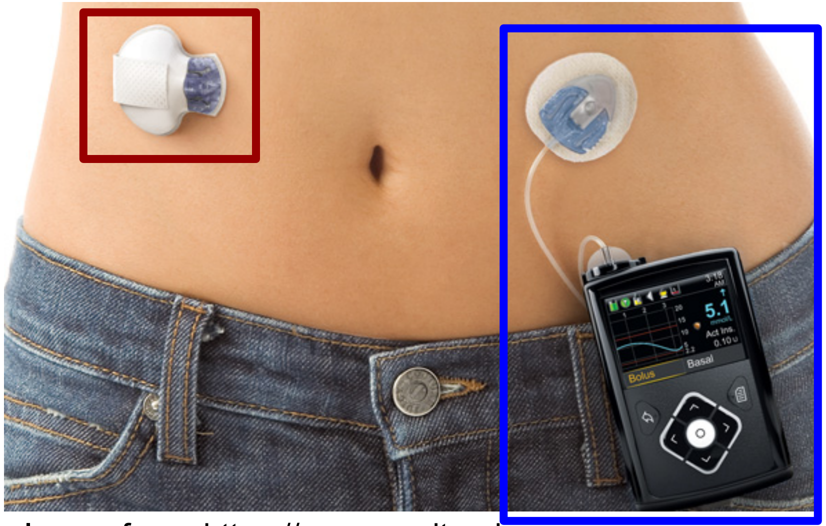


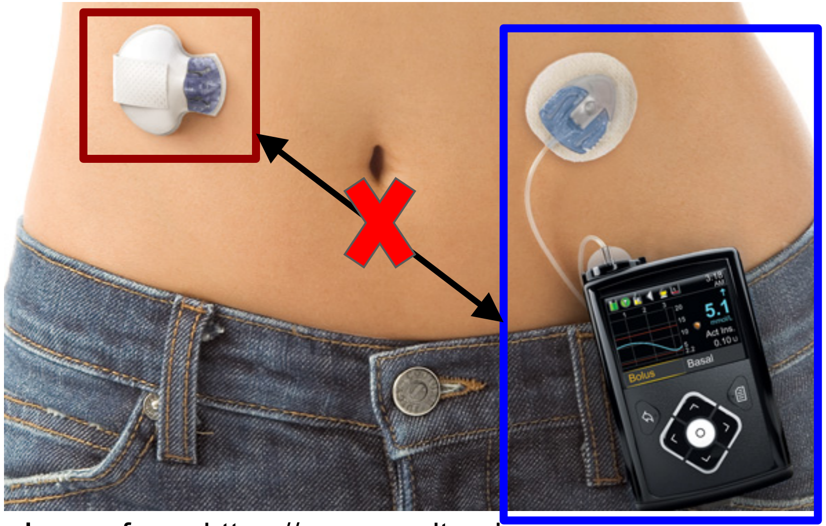
Image from: https://www.medtronic-diabetes.com.au/pump-therapy/what-is-insulin-pump-therapy

Insulin pump delivers two kinds of insulin:

- **Bolus**: high, on-demand dose to cover meals
- **Basal:** to cover demand outside meals

Continuous Glucose Monitor (CGM) detects sugars levels under the skin, a measure of **blood glucose (BG)**

T1D therapy - limitations



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- Bolus is manually set by the patient → **danger of wrong dosing**
- Pump and CGM don't communicate with each other

Closed-loop control, AKA Artificial Pancreas (AP)

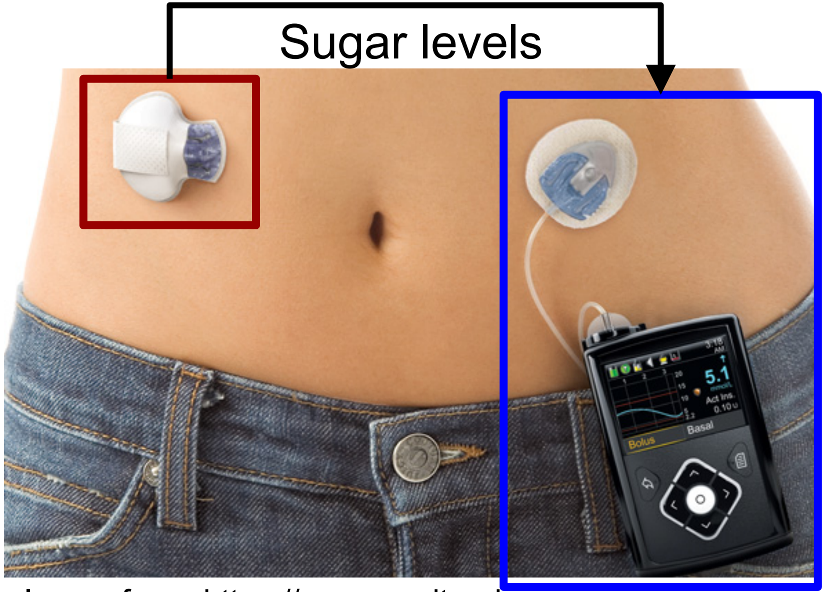


Image from: https://www.medtronic-diabetes.com.au/pump-therapy/what-is-insulin-pump-therapy

Challenges:

- CGM is a “derived” measure of BG
- Disturbances (Meal and Exercise)

Closed-loop control, AKA Artificial Pancreas (AP)

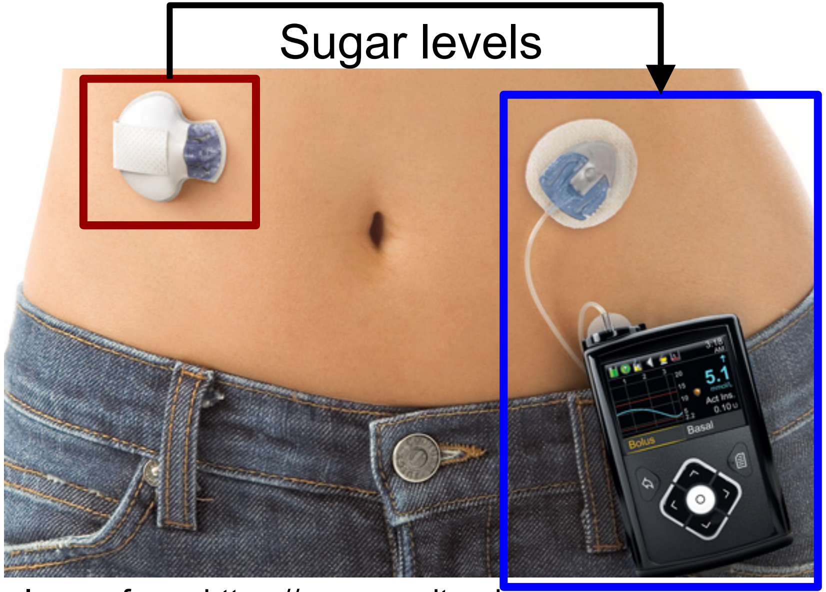


Image from: https://www.medtronic-diabetes.com.au/pump-therapy/what-is-insulin-pump-therapy

- Disturbances (Meal and Exercise)

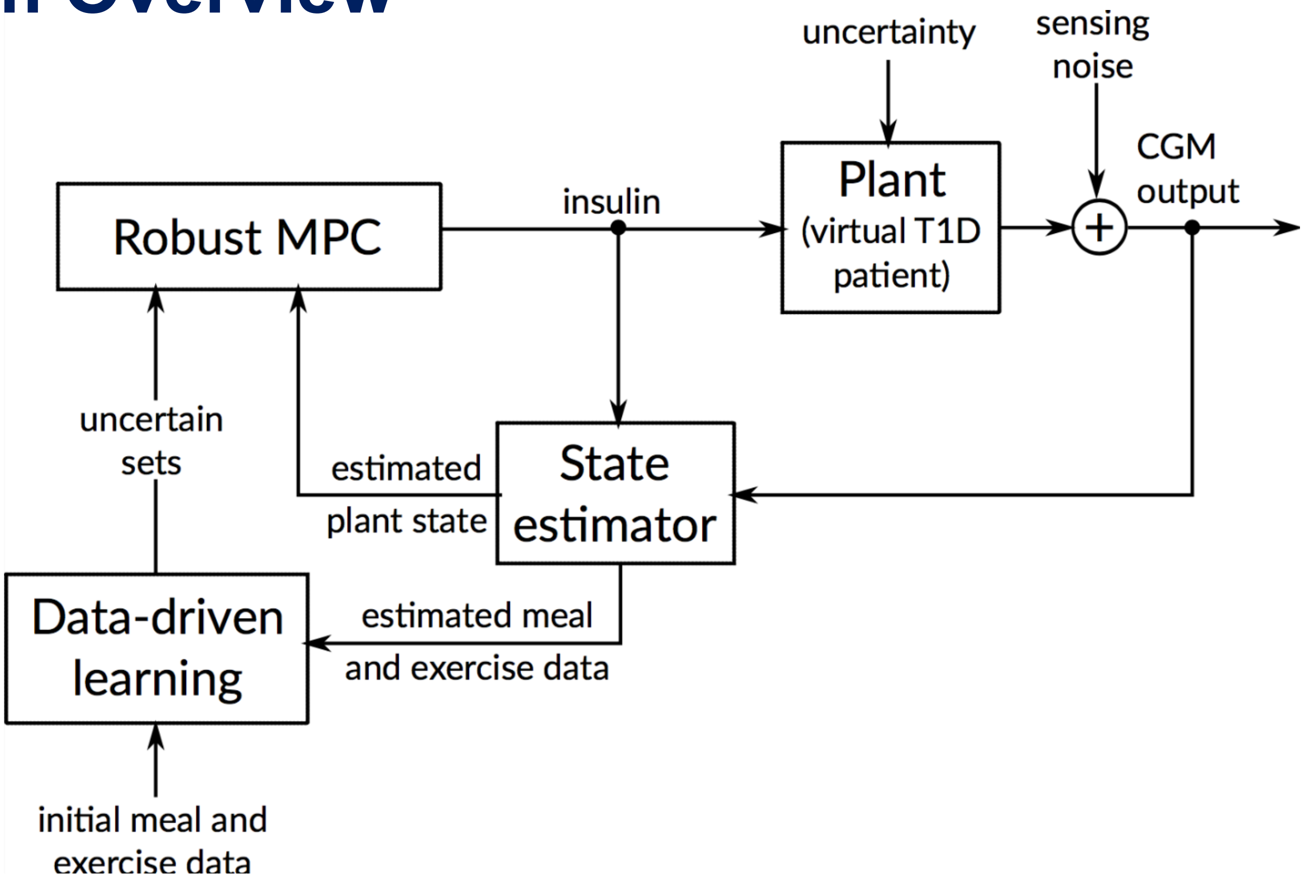
- Only controls basal insulin
- Meals are still announced

Our contribution

A **data-driven robust model predictive control (MPC)** design for the AP:

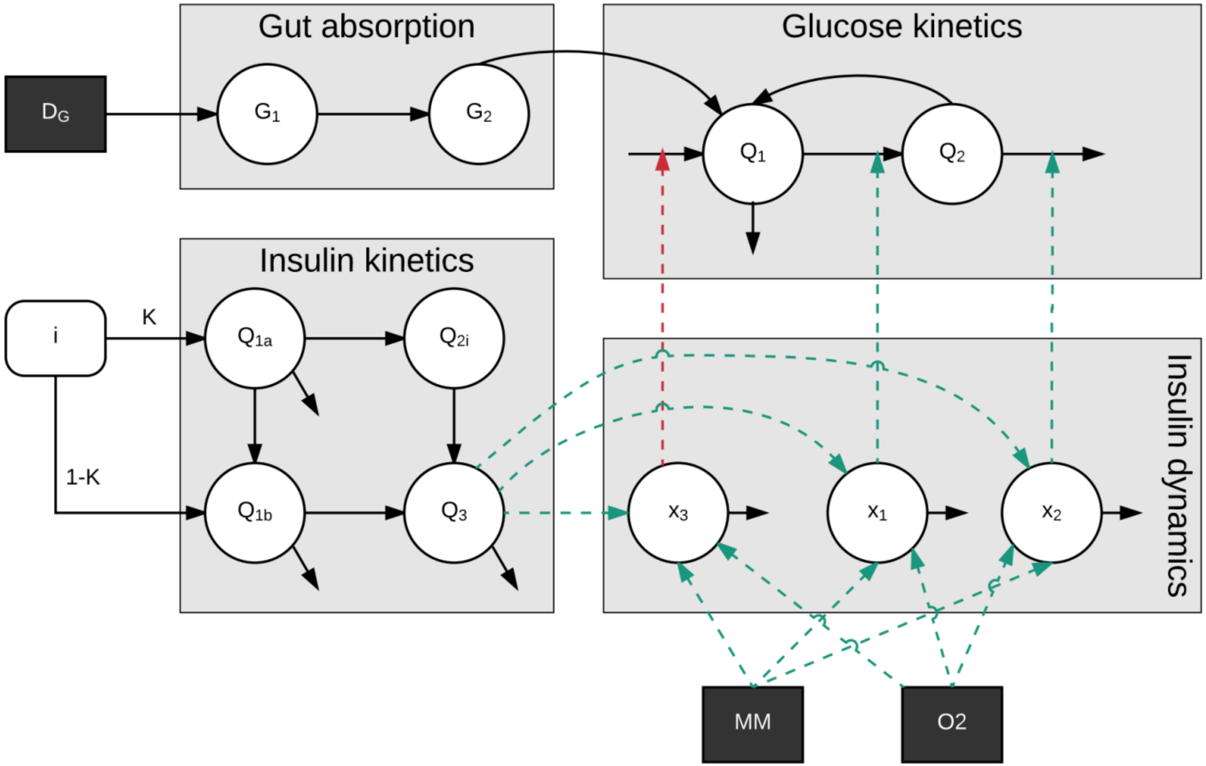
- Closed-loop control of **both basal and bolus insulin**
- Handles uncertainty by **learning from data**
- Accurate **state estimation from CGM measurements**

System Overview



Simulation Model

Plant
(virtual T1D patient)



Hovorka et al., Physiol. Meas. 25 (2004) 905–920
Wilinska et al., Journal of Diabetes Science and Technology 4 (1), 2010
Resalat et al., EMBC 2016

Simulation Model

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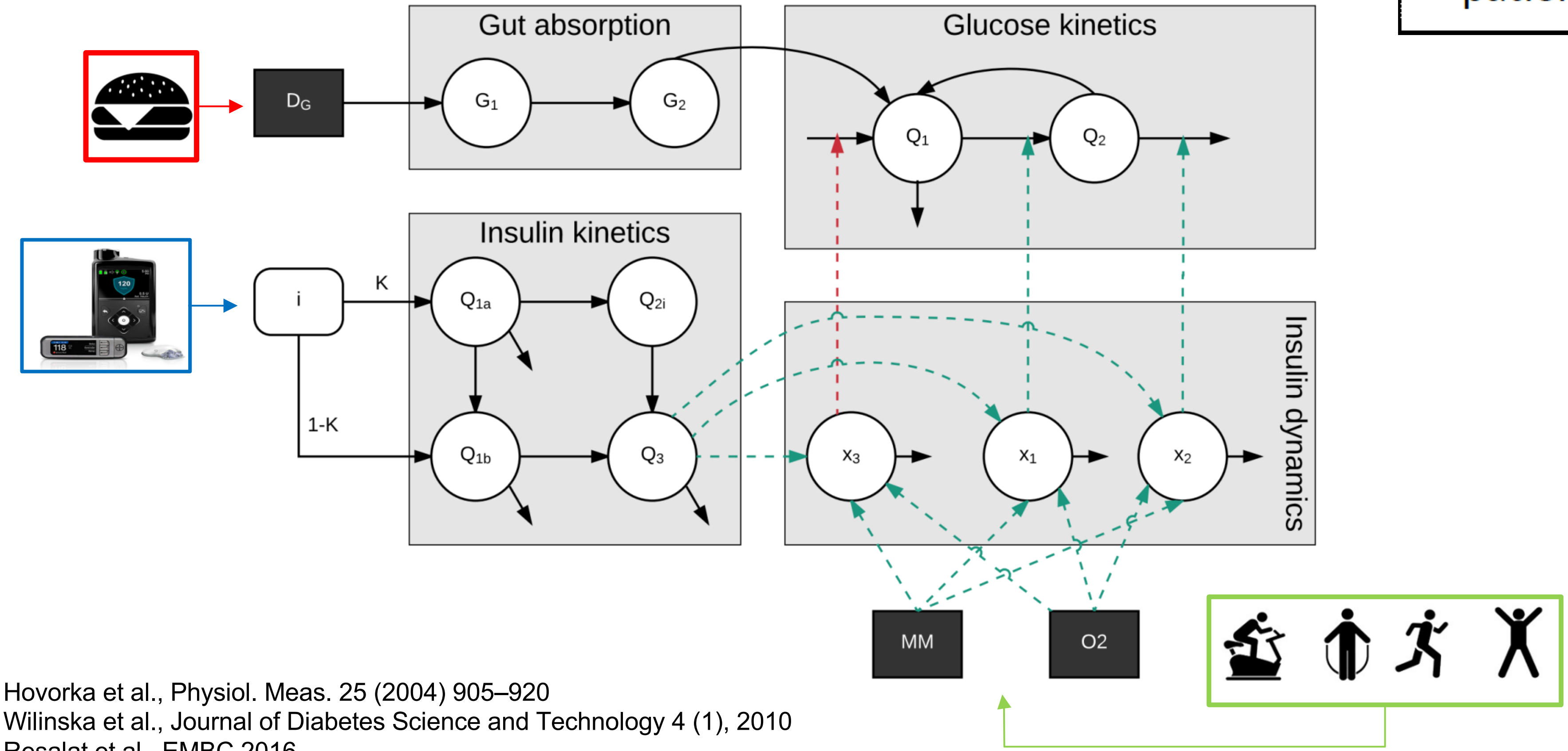

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Uncertainty Sets

Data-driven learning

Learn from data the possible realization of uncertainty parameters

$$\mathcal{U} = \begin{pmatrix} \mathbf{u}^t \\ \mathbf{u}^{t+1} \\ \vdots \\ \mathbf{u}^{t+N_p-1} \end{pmatrix}$$

Model with spatial (among meal and exercise) and temporal (among times) correlation

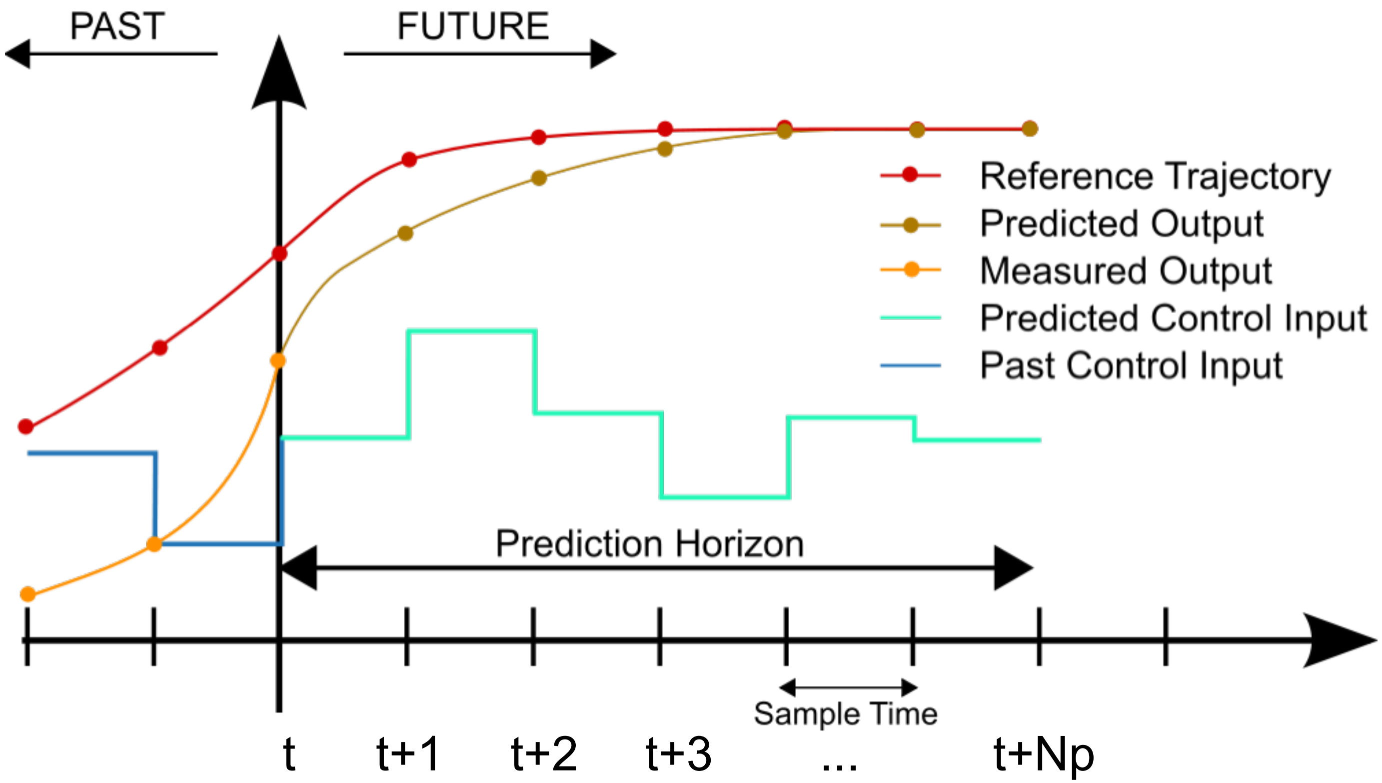
$$\mathcal{U} = \{\underbrace{\boxed{\hat{\boldsymbol{\mu}}}}_{\text{mean}} + \underbrace{\boxed{\mathbf{C}^T}}_{\text{covariance}}\mathbf{w}\}$$

Bertsimas et al., *arXiv:1401.0212* (2013)

Robust Model Predictive Control

Robust MPC

MPC:

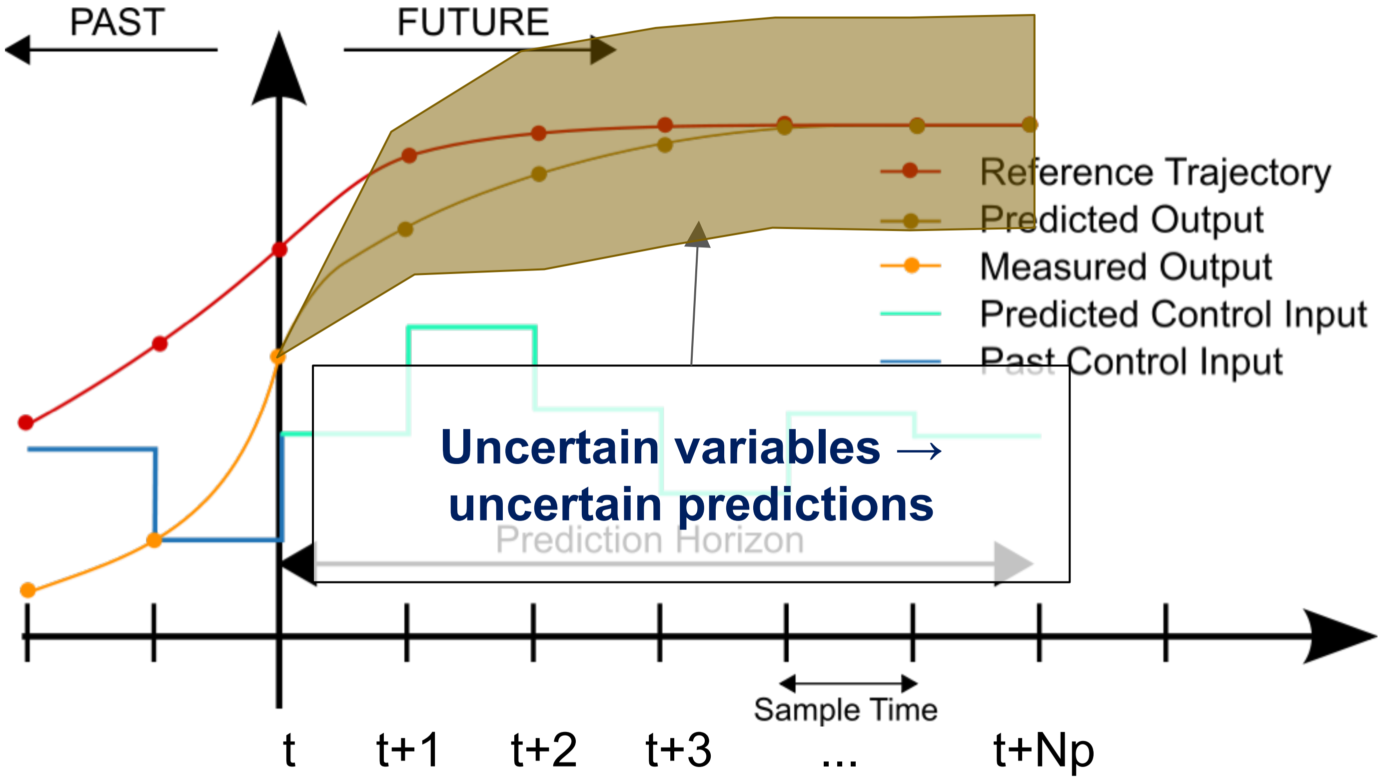


Picture from:https://en.wikipedia.org/wiki/Model_predictive_control

Robust Model Predictive Control

Robust MPC

Robust MPC:



Uncertain variables → uncertain predictions

Picture from:https://en.wikipedia.org/wiki/Model_predictive_control

Robust Model Predictive Control

Robust MPC

Find the insulin therapy at time t, t+1,... that minimizes the worst case performance w.r.t. uncertainty parameters

$$\boxed{\min_{\iota^t,\dots,\iota^{t+N_c-1}}}\boxed{\max_{\mathbf{u}^t,\dots,\mathbf{u}^{t+N_p-1}}}\boxed{\sum_{k=1}^{N_p} d(t+k)}+\boxed{\beta \cdot \sum_{k=0}^{N_c-1} (\Delta\iota^{t+k})^2}$$

Performance: combination of distance from target trajectory and step-wise discrepancy of control strategy

State Estimation

State
estimator

We designed a **Moving Horizon Estimator (MHE)**:

- "Estimation *a la* MPC": uses a model to minimize distance between predicted and actual measurements, and between predicted and estimated states over a moving window of length N

- It works also as a **meal estimator:** estimates the most-likely uncertainty parameter values

$$\min_{\mathbf{x}^{t-N},\ldots\mathbf{x}^{t},\boxed{\mathbf{u}^{t-N},\ldots,\mathbf{u}^{t}}} \boxed{\mu \cdot \|\mathbf{x}^{t-N} - \hat{\mathbf{x}}^{t-N}\|^2} + \boxed{\sum_{k=t-N+1}^{t} \frac{\|v^k\|^2}{q^k}}$$

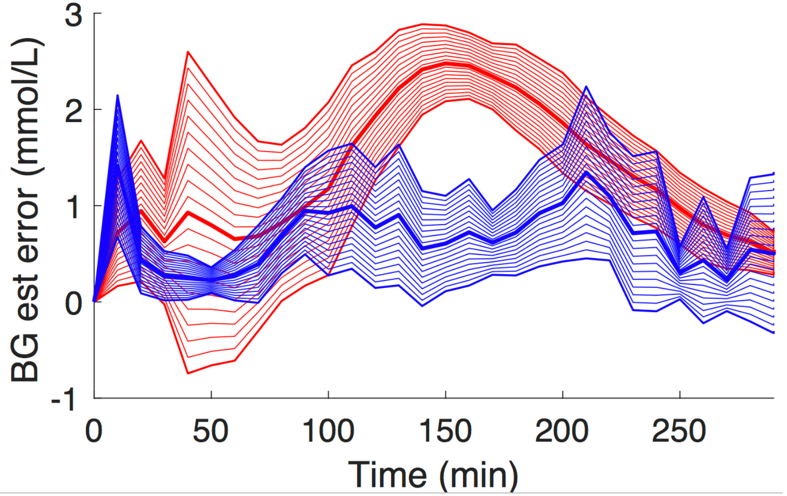
Rao et al., IEEE Trans. Automatic Control 48 (2), 246-258, 2003

State Estimation

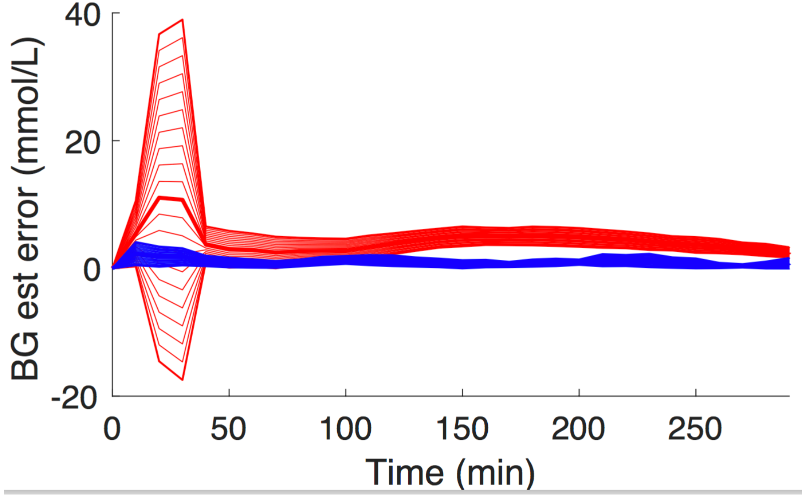
State estimator

MHE outperforms Extended Kalman filter

Low sensor noise



High sensor noise



Evaluation

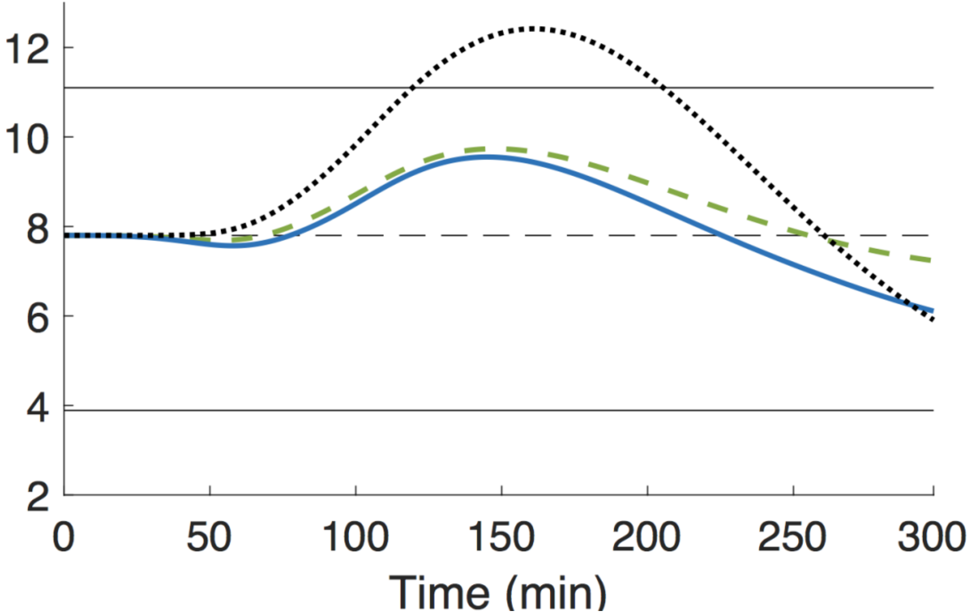
Robust controller compared with

- **Perfect controller:** with exact knowledge of uncertainty parameters and full state observability (no state estimation errors)

- **Only basal controller:** simulates the behavior of hybrid closed-loop insulin pumps with only automatic control of basal insulin (bolus is manual)

Scenario 1 - Meals as expected

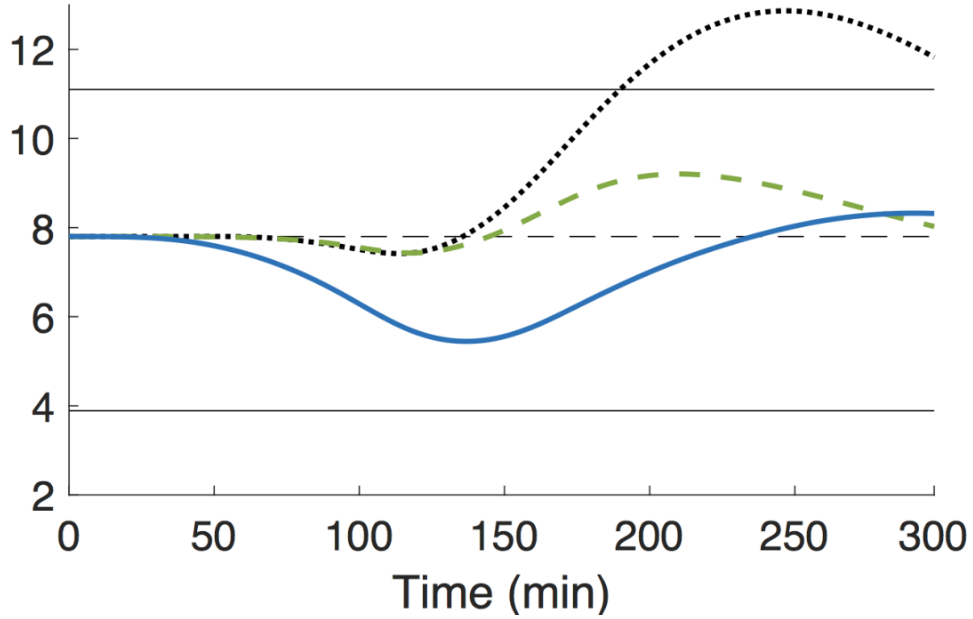
- Start: 60 ± 30 min; total carbs: 60 ± 18 g
- **Situation where uncertainty set is accurate**



	T hypo	T normal	T hyper
Perfect	0%	99.69%	0.31%
Only basal	1.6%	69.4%	29%
Robust	**0.51%**	**97.7%**	**1.79%**

Scenario 2 - Unexpected delays

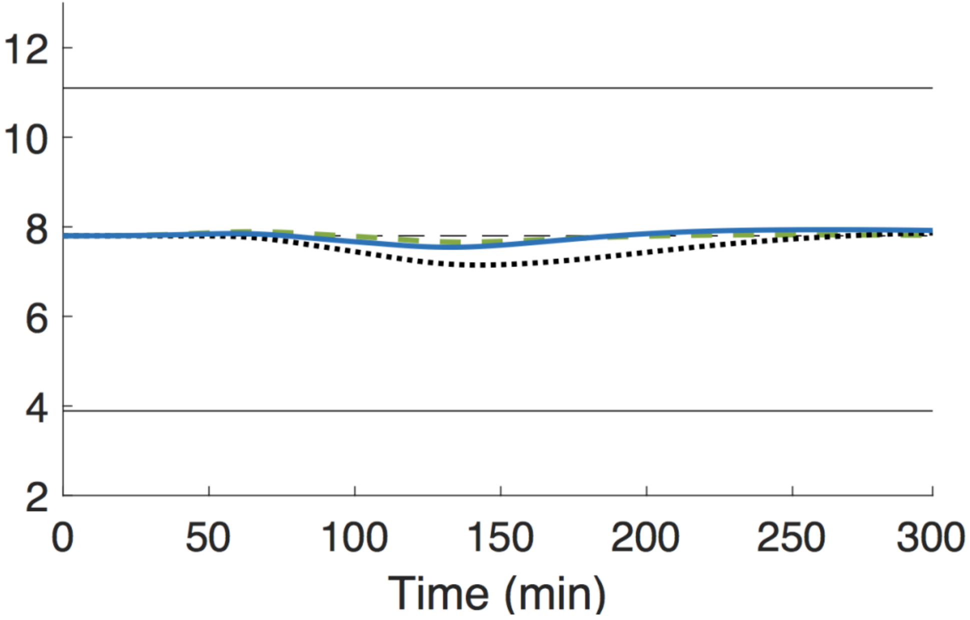
- Same as before, but start time is constantly delayed of 1 h
- **Situation where uncertainty sets are not accurate**



	T hypo	T normal	T hyper
Perfect	0%	100%	0%
Only basal	**0%**	67.25%	32.75%
Robust	0.79%	**99.03%**	**0.18%**

Exercise

- We reproduce a 1h exercise (30 min intense, followed by 30 min moderate)



	T hypo	T normal	T hyper
Perfect	0%	100%	0%
Only basal	0%	100%	0%
Robust	0%	100%	0%

Summary

- Robust MPC design for AP that well supports meal disturbances
- Based on deriving uncertainty sets from patient data
- Towards fully closed-loop diabetes therapy

Ongoing and future work

- Longer scenarios (1, 2, 7, … days)
- Data from CDC’s NHANES database
- More tractable formulation (linearization, convex programming, simpler model)
- More advanced patient behavioral model