Georgia Tech School of Physics

CyberCardia

Real Time Modeling of Cardiac Tissue

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CHAOS Lab

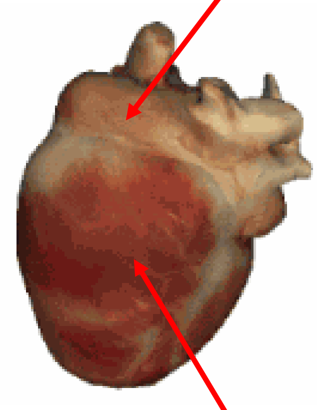
Complex Heart Arrythmias and other Oscillating Systems

- We need to model 2D and 3D hearts

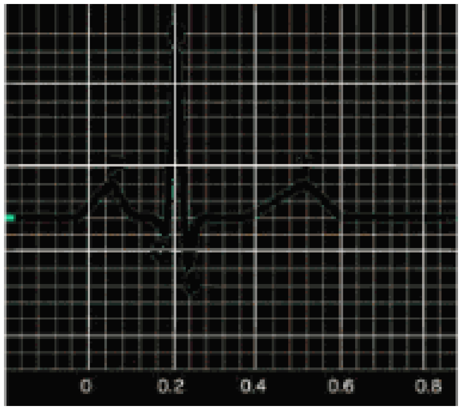
- Reproduce recordings used by machines (Electrode and ECGs)

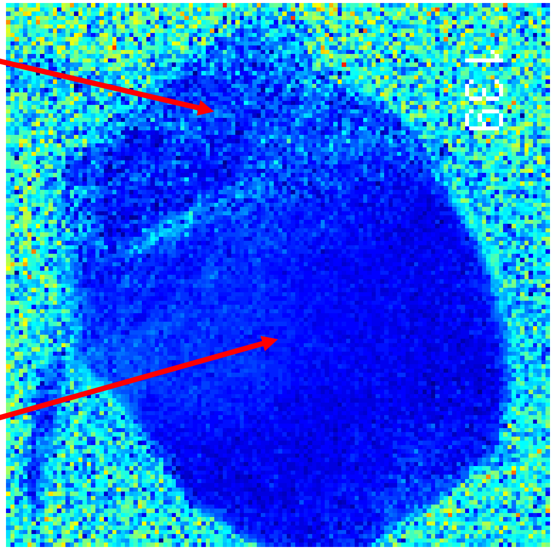
What is the ECG

Atria



ECG

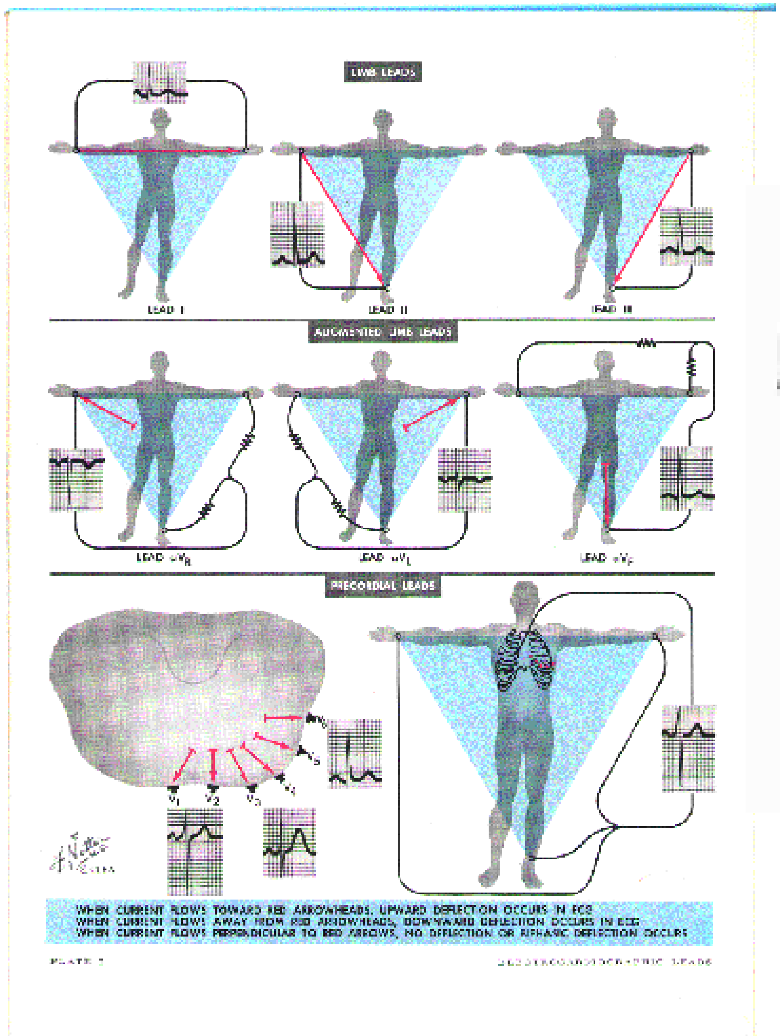


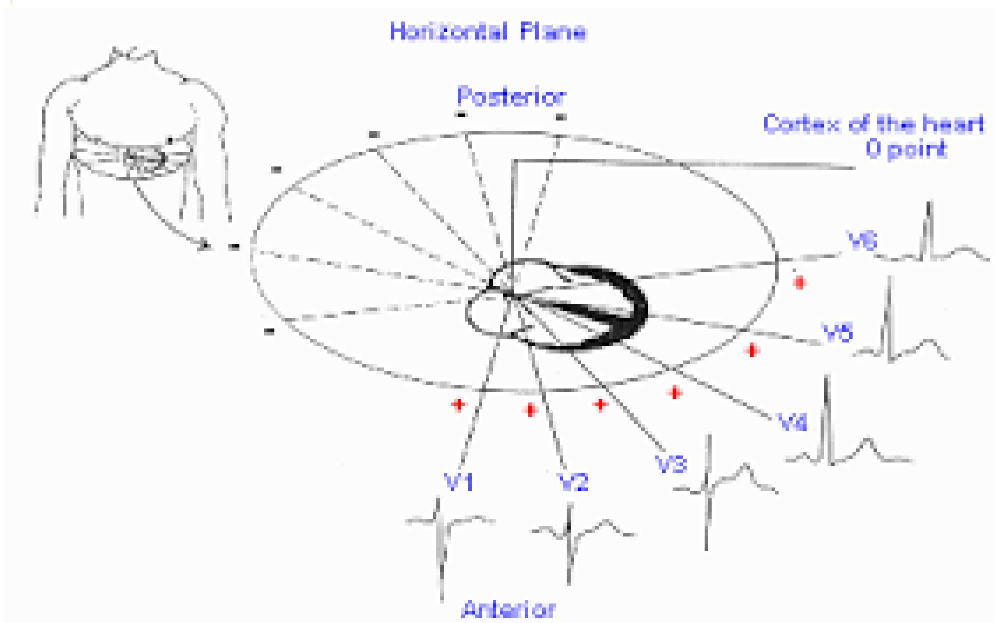


Ventricles

Using optical mapping.
Rabbit heart

ECG leads.


LIMB LEADS
LEAD I
LEAD II
LEAD III
AUGMENTED LIMB LEADS
PRECORDIAL LEADS
WHEN CURRENT FLOWS TOWARD RED ARROWHEADS, UPWARD DEFLECTION OCCURS IN ECG
WHEN CURRENT FLOWS AWAY FROM RED ARROWHEADS, DOWNWARD DEFLECTION OCCURS IN ECG
WHEN CURRENT FLOWS PERPENDICULAR TO RED ARROWS, NO DEFLECTION OR BIPHASIC DEFLECTION OCCURS


Horizontal Plane
Posterior
Cortex of the heart
0 point
V6
V5
V4
V3
V1
V2
Anterior

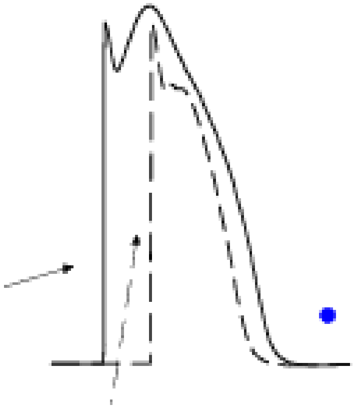
Things to Note:

Direction of activation gives QRS (given by Purkinje activations)

T-wave is given by the irregular wave back

Purkinje Fibers

Regions of cells with different durations



Multi dimensional system (from 0 to 3D)

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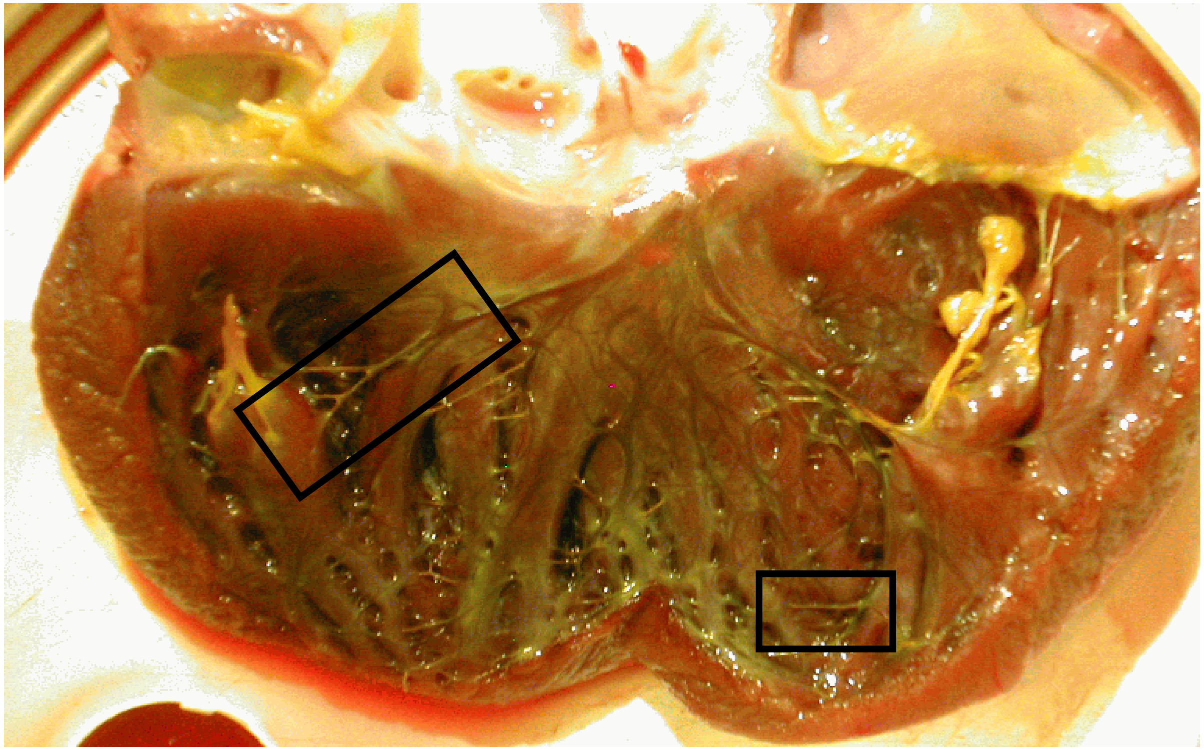
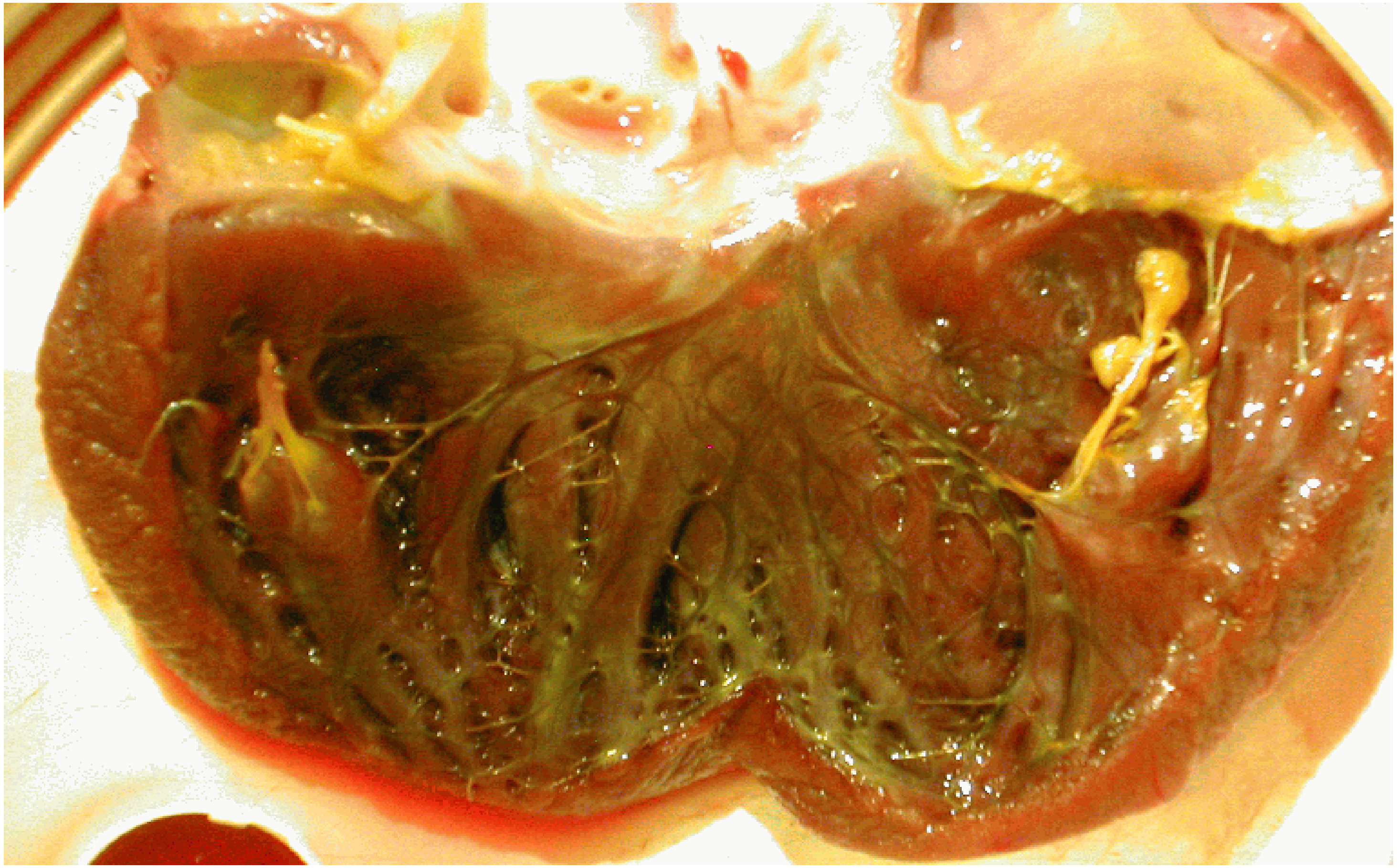
Open heart and Purkinje network

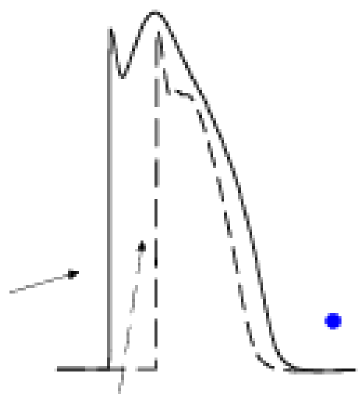


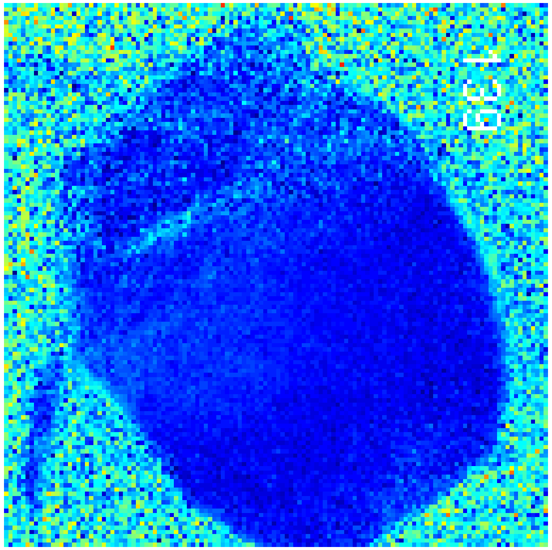
Multi dimensional system (from 0 to 3D)

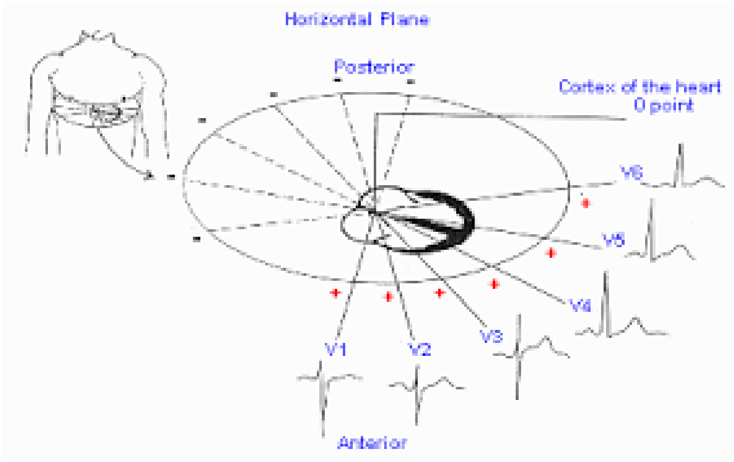
Open heart and Purkinje network



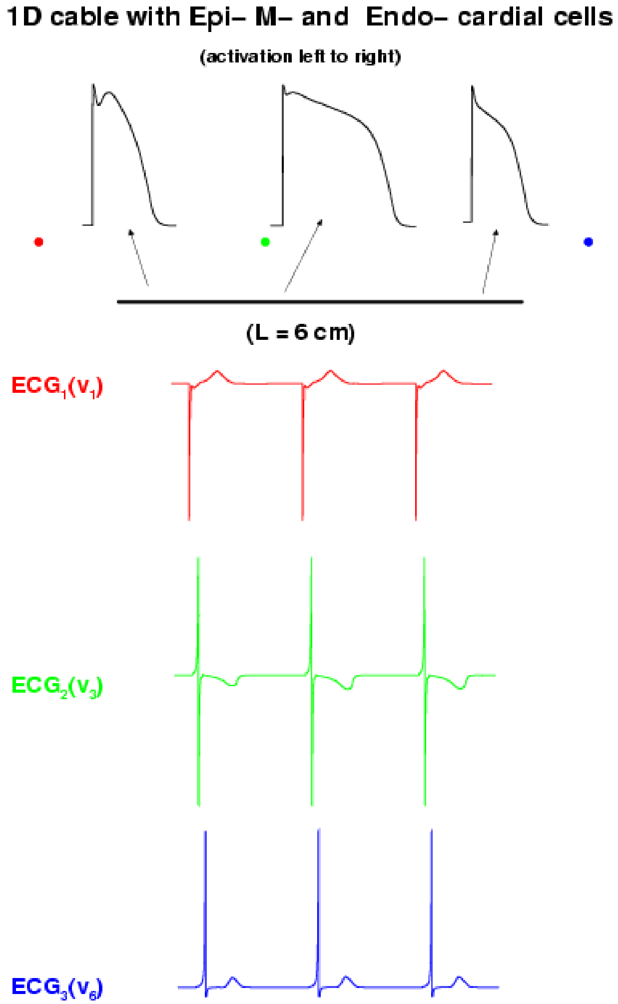
T-wave is given by the irregular wave back







How do we implement ECG?



ECG from simulations and Exp.

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$$V_m(\mathbf{r},t) = u_i(\mathbf{r},t) - u_e(\mathbf{r},t), \tag{1}$$

where u_i and u_e are the intra- and extracellular myocardial potentials at point r and time t, respectively. Let the current densities be of the form $\mathbf{j}_\mu = -g_\mu \nabla u_\mu$ for the intra-, extracellular, and extracardiac regions, respectively. There are no current sources or sinks within the body, so the continuity equation requires

$$\begin{cases} 0 = \nabla \cdot (\mathbf{j}_i + \mathbf{j}_e)\,\big|_{r\in\Omega_H} \\ 0 = \nabla \cdot \mathbf{j}_0\big|_{r\in\Omega_0}. \end{cases} \tag{2}$$

And the flux continuity across the boundary between the heart and extracardiac medium requires

$$\begin{cases} u_e = u_i \\ \mathbf{j}_0 \cdot \hat{\mathbf{n}} = (\mathbf{j}_i + \mathbf{j}_e) \cdot \hat{\mathbf{n}} \end{cases} \tag{3}$$

along the boundary $\partial\Omega_H$. Within the heart, transmembrane potential differences $V_m(\mathbf{r},t)$ provide an equivalent cardiac source when related as

$$\mathbf{j}(\mathbf{r}) = -g_i \nabla V_m, \tag{4}$$

where g_i is the intracellular membrane conductance. We may then express the total current density as a sum including both the transmembrane potential V_m and the total electrostatic potential $\varphi(\mathbf{r},t)$

$$\mathbf{j} = -\sigma_0 \nabla\varphi - g_i \nabla V_m. \tag{5}$$

Since the divergence of the total current density is zero according to equation 4,

$$0 = -\nabla(\sigma_0 \nabla\varphi) - \nabla(g_i \nabla V_m), \tag{6}$$

It is possible to write a Poisson equation for the electrostatic potential in terms of the transmembrane potential

$$\nabla^2\varphi(\mathbf{r}) = -\frac{g_i}{\sigma_0}\nabla^2 V_m. \tag{7}$$

$$\begin{aligned} \frac{g_i}{\sigma_0}\varphi(\mathbf{r}) &= -\int_{\Omega_H} d^3r' \left\{\nabla'\left[G\,\nabla' V_m\right] - G\,\nabla'^2 V_m\right\} \\ &= \int_{\Omega_H} d^3r' G\,\nabla'^2 V_m - \oint_{\partial\Omega_H} d\mathbf{S}\cdot G\,\nabla' V_m \end{aligned} \tag{9}$$

Since the ECG probe is located external to the heart, $\mathbf{r} \notin \Omega_H$. By the Neumann boundary conditions imposed upon the Green's function, the surface term is zero if we take the approximation that the conducting medium has equal anisotropy ratios, or $g_e \propto g_i$. For an infinite and homogeneous extracardiac medium Ω_0, we have the Green's function

$$G(\mathbf{r};\mathbf{r}') = \frac{1}{4\pi\,|\mathbf{r}-\mathbf{r}'|}. \tag{10}$$

Substitution of this Green's function, assuming equal anisotropy ratios, and utilizing the divergence theorem, we arrive at the integral formulation for the electrostatic potential at point $\mathbf{r}$ in terms of the transmembrane potential

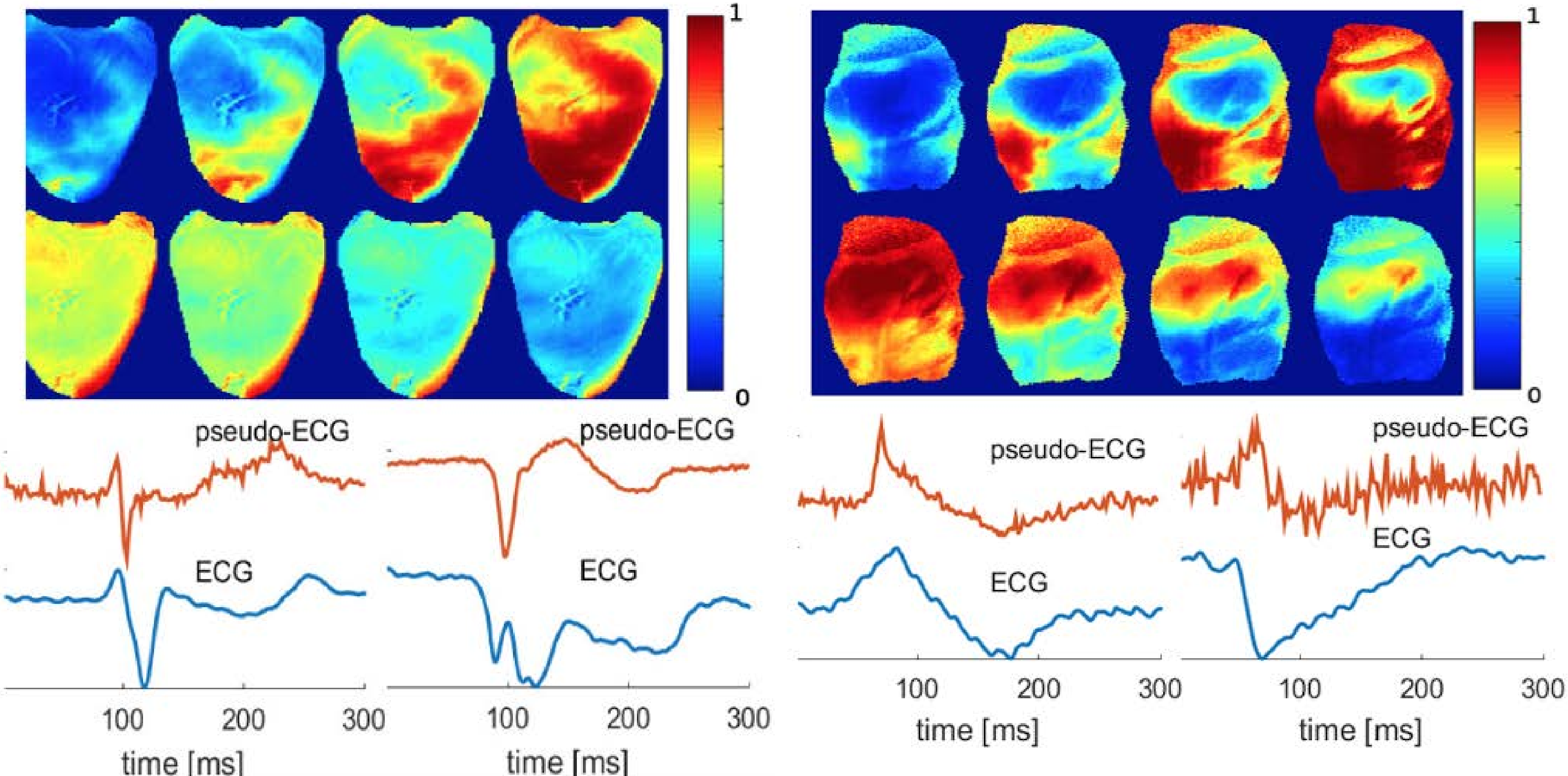
$$\varphi(\mathbf{r}) = \frac{g_i}{4\pi\sigma_0}\int_{\Omega_H} d^3r' \frac{\nabla'^2 V_m(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}. \tag{11}$$

Since the dimensional scales associated with the transmembrane potential difference and ECG amplitude are known, the important characteristic involved in OM-ECG calculation is the relationship between its amplitude and time. Assuming the transmembrane potential has the form $V_m(\mathbf{r}) = V_m(x,y)$, with constant V_m along the z-direction, we have the proportionality

$$\varphi(\mathbf{r}) \propto \int d^2r' \frac{\nabla'^2 V_m(x,y)}{|\mathbf{r}-\mathbf{r}'|}. \tag{12}$$

- Reconstructed ECG (from experiment, or numerical)

$$\varphi(\mathbf{r}) = \frac{g_i}{4\pi\sigma_0} \int_{\Omega_H} d^3r' \frac{\nabla'^2 V_m(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}.$$



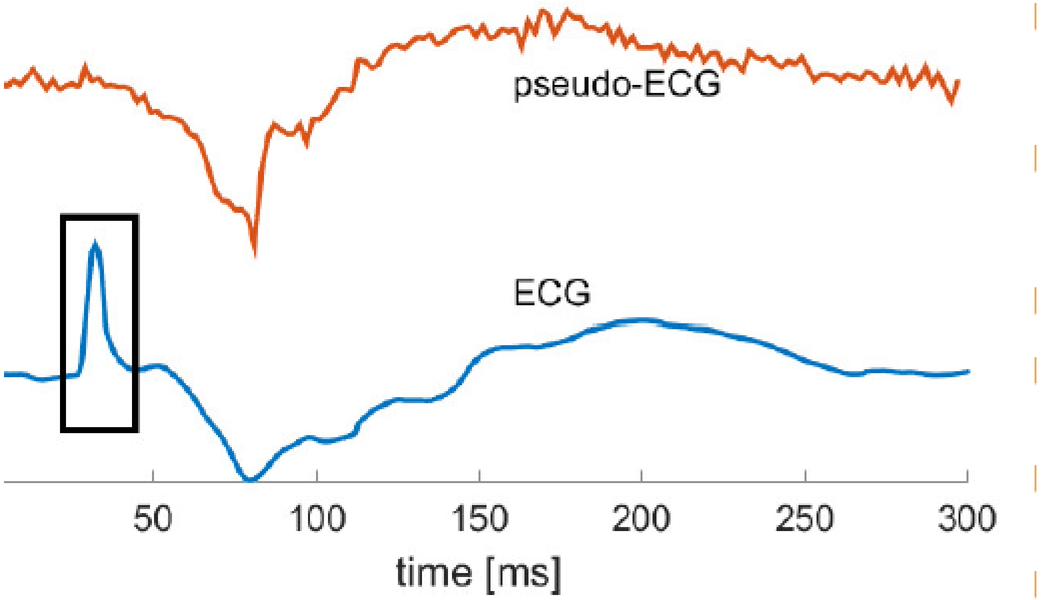
Normal

ischemic

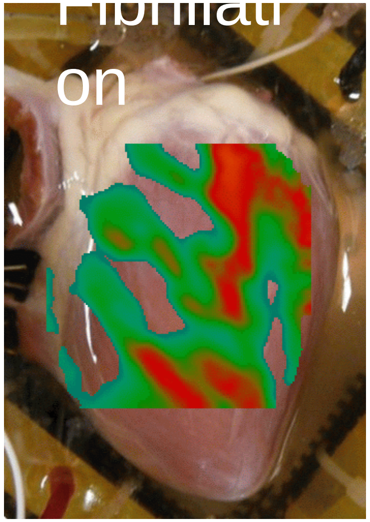
- Reconstructed ECG (from experiment, or numerical)

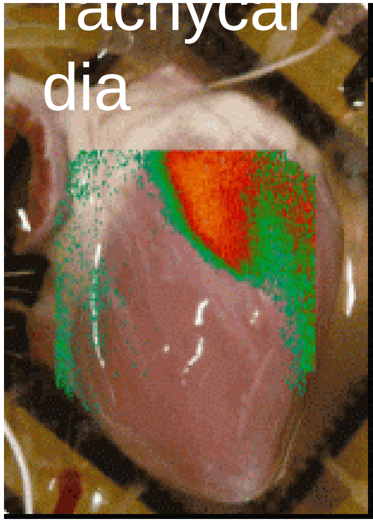
$$\varphi(\mathbf{r}) = \frac{g_i}{4\pi\sigma_0} \int_{\Omega_H} d^3 r' \frac{\nabla'^2 V_m(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}.$$

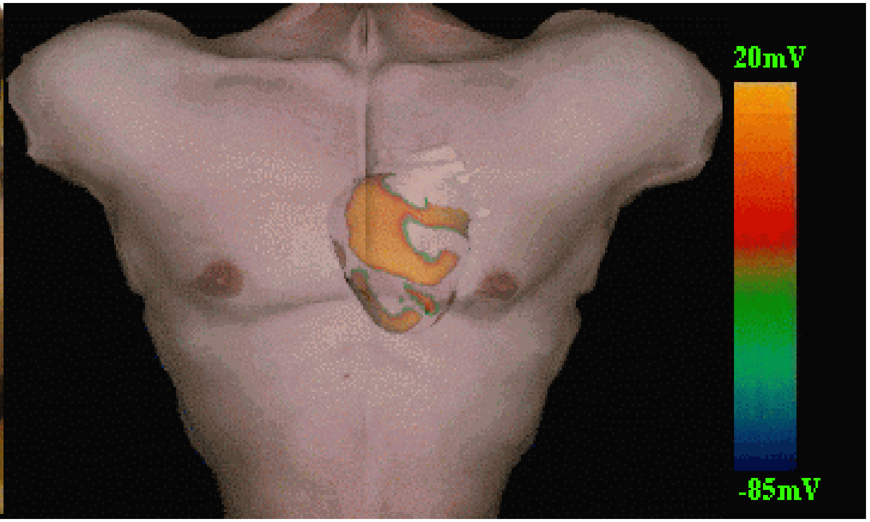
Advantage!



Real time Simulations of this:

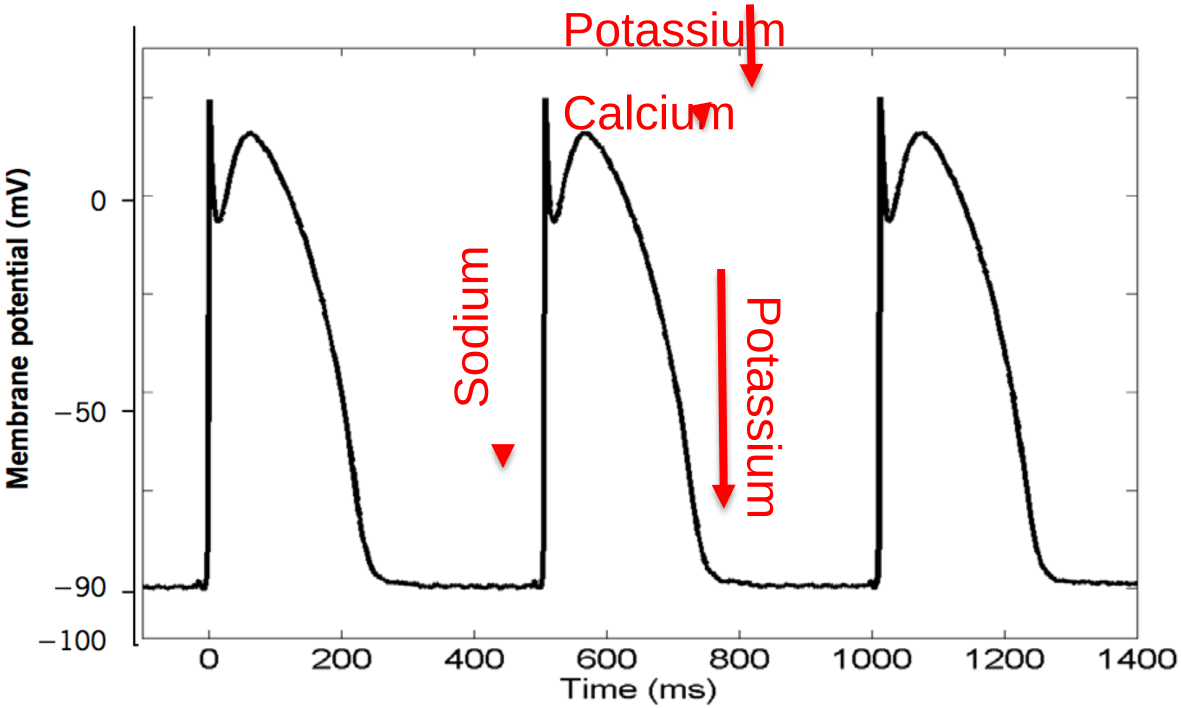


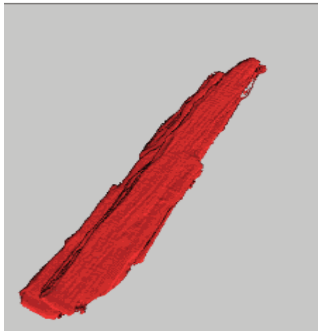




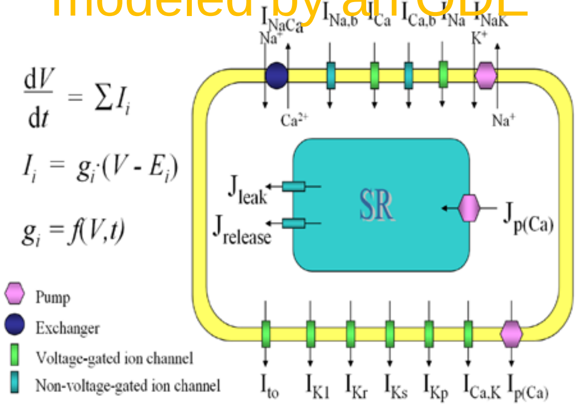
Simulations

2D Simulations



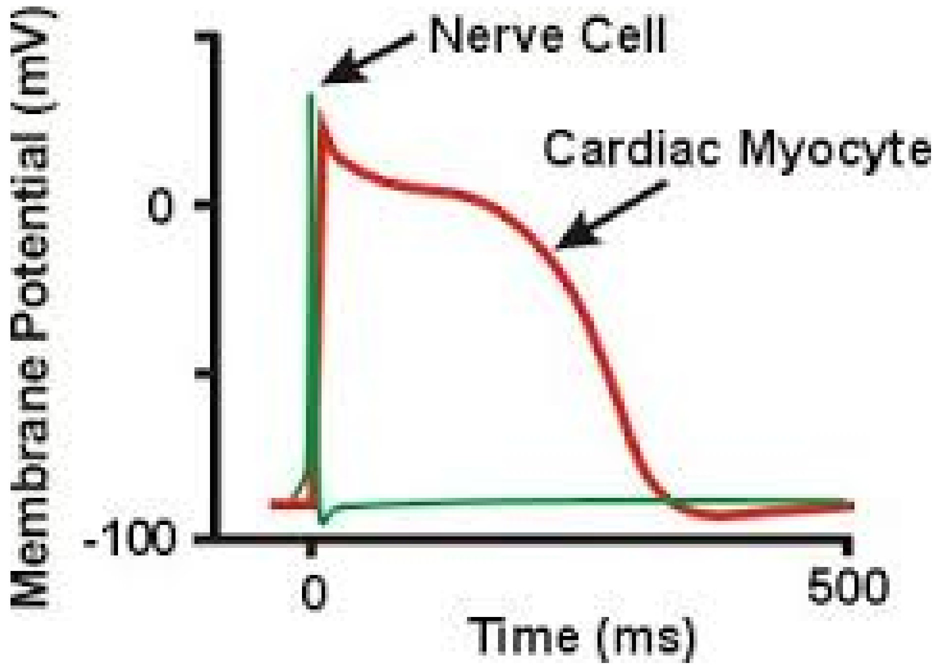


Mathematically, each current can be modeled by an ODE



The more complex the model the more equations to solve

Simulations



Stiff ODEs

Euler Method

Time for the upstroke ~5 mseconds !!

the

i.e. 1 second requires 10,000 iterations!

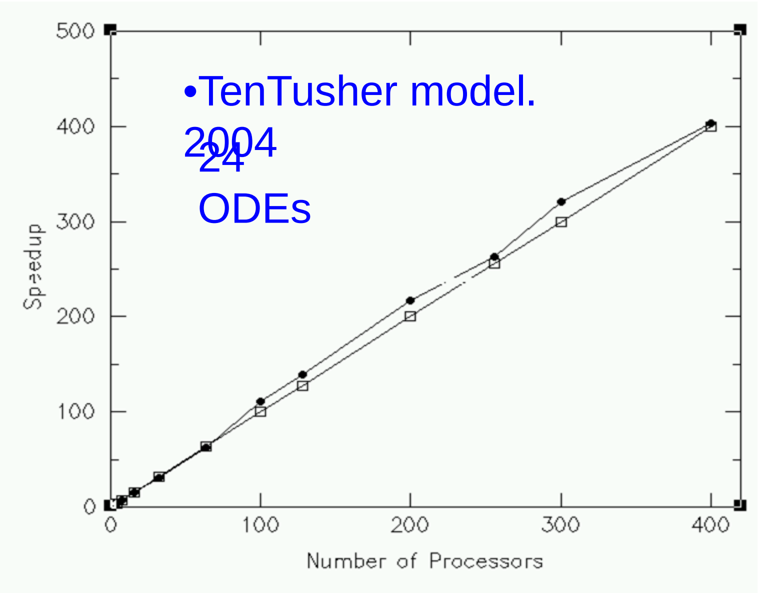
Simulations

1 second requires 10,000 iterations!

Each cell (4 to 24) ODEs

Number of cells in tissue?

Millions!



have to Solve:

10×10^{11} ODEs per second