Query Checking for Linear Temporal Logic

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Verification

Model checking: "Does model $\mathcal{M}$ satisfy property ϕ?"

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Discovery

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Query Checking

For a model $\mathcal{M}$ and a property *template* $\phi[x]$, what is a solution c for x such that $\mathcal{M}$ satisfies $\phi[x := c]$?

Example

"What conditions must be met for the car to eventually stop?"

- Restrict our properties of interest to LTL formulas.

Outline

1. Linear Temporal Logic
2. LTL Formulas as Automata
3. LTL Model Checking
4. LTL Query Checking
5. Edge Shattering

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Some Related Work

- Query checking originally for CTL (Chan CAV 2000, Gurfinkel TSE 03)
- Earlier work in LTL query checking (Chockler HVC 2010)

Linear Temporal Logic (LTL)

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- **G** *sunny* – It is always sunny (in Philadelphia).

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- $\mathbf{X}\, \mathit{rain}$ – Tomorrow will be rainy.
- $\mathit{red}\, \mathbf{U}\, \mathit{green}$ – The traffic light is red until it is green.
- $\mathbf{G}\, \mathit{sunny}$ – It is always sunny (in Philadelphia).
- $\mathbf{G}(\mathit{button} \rightarrow \mathbf{F}\, \mathit{stop})$ – Whenever the button is pressed, the machine eventually stops.

LTL formulas as automata

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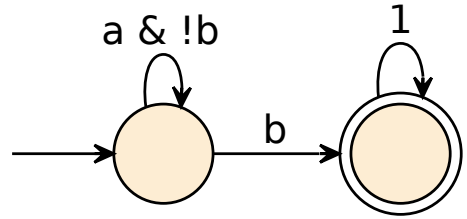
$a \, \mathbf{U} \, b \quad \Longrightarrow$



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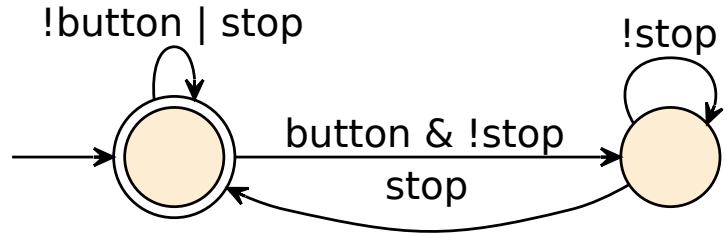
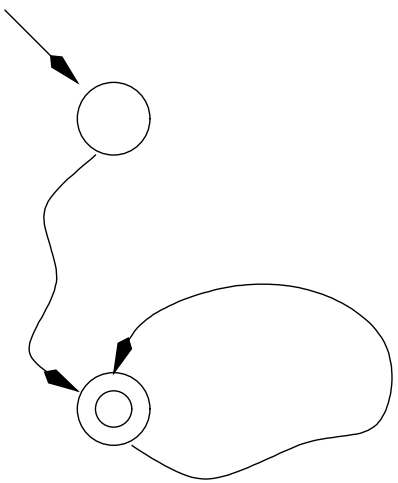


- As automata, facilitates LTL model checking when using similar style automata for data.
- Want to work in this representation to discover new properties.

LTL Model Checking

Automaton-based methods (Vardi, Wolper 86)

1. Take model $\mathcal{M}$, convert to an automaton representation B_M.
2. Take LTL formula ϕ, compute automaton for $\neg\phi$: $B_{\neg\phi}$.
3. Compute a composed automaton $B_c = B_M \cap B_{\neg\phi}$
4. Determine if $L(B_c) = \emptyset$



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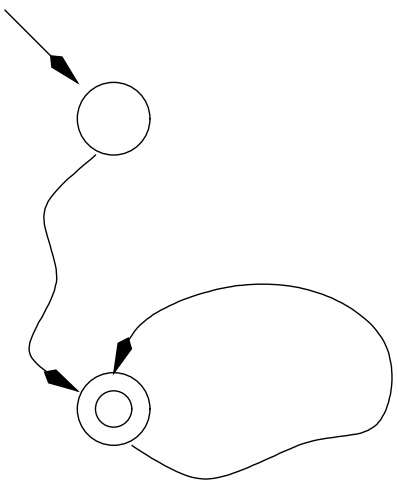

LTL Templates from LTL Formulas

- An LTL template contains some unknown.
- Relax one part of an LTL formula to be variable (placeholder).

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- Can we modify existing LTL model checking approach to help us perform discovery?

Automaton Based Query Checking

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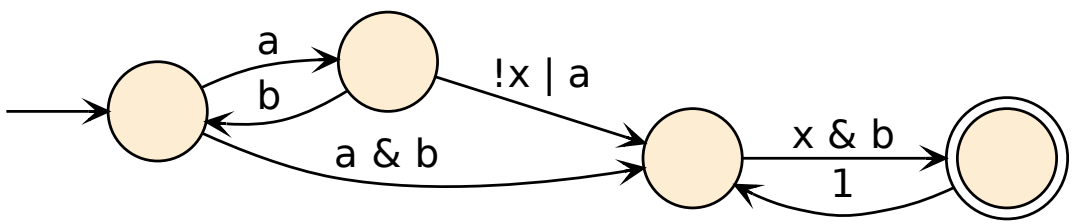
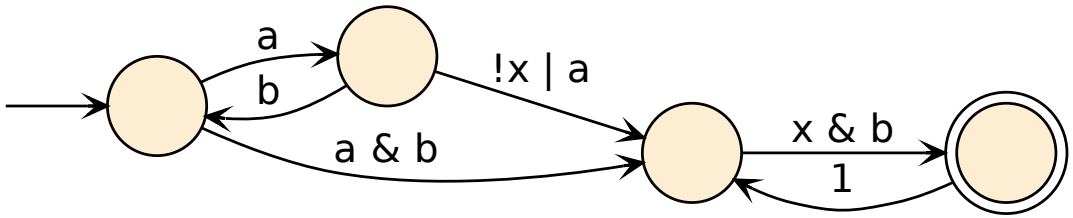
From LTL Model Checking:

- Compute a composed automaton $B_c = B_M \cap B_{\neg\phi}$
  - What do the transition labels look like on B_c?
- Determine if $L(B_c) = \emptyset$
  - How is this determined given the new edge labels?
  - The choice of grounding for the variable can invalidate edges



Büchi Propositional Automata



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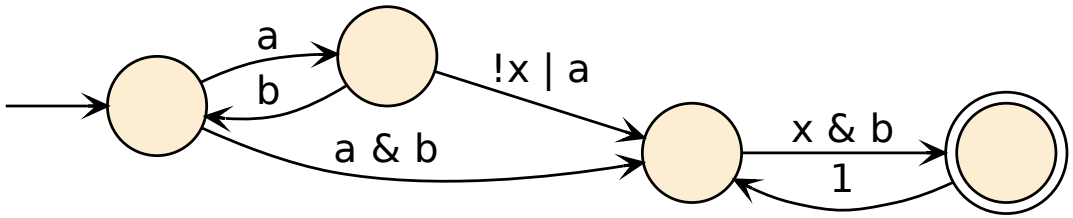

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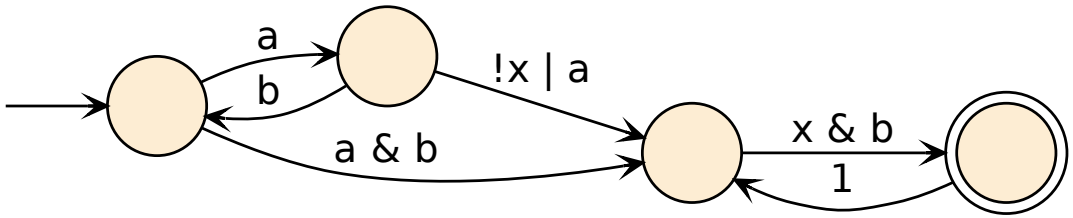

- Automaton representation of infinite length languages.
- Standard product composition of Büchi automata uses single alphabet symbols.
- Because an LTL template contains a variable, labels along transitions of the composition Büchi will contain propositional formulas as well.

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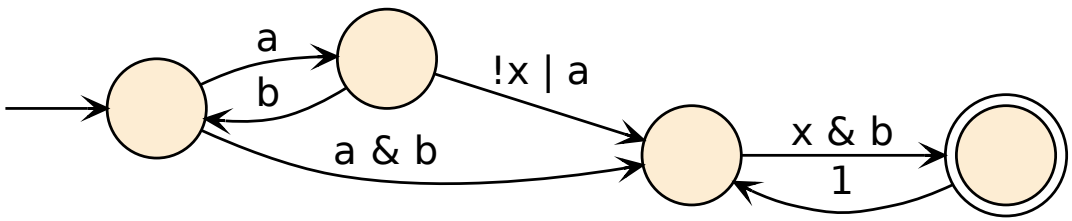

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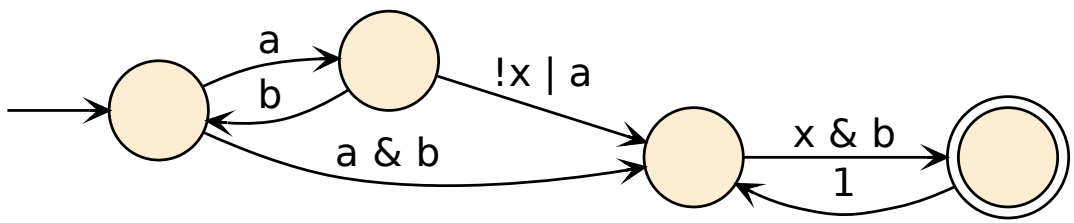

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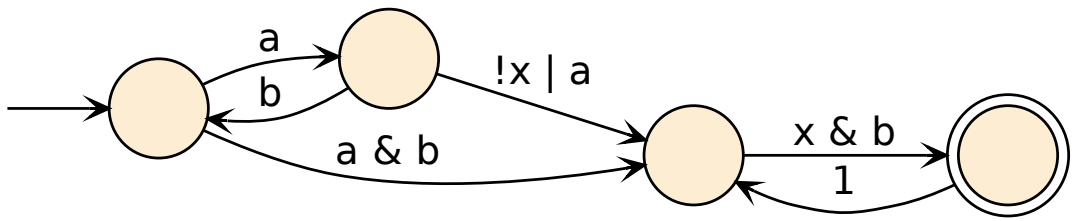

- Choice of edge set $\longrightarrow$ constraints on assignments for x
- Resolve all constraints to produce solutions for x.

Conclusion

Summary

- Developed automaton-based approach for LTL query checking
- "Shattering condition" problem yielded interesting side results

Ongoing & Future Work

- Expanding selection of datasets for evaluation
- Allowing for multiple variables/larger uncertainty

Thanks!

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